A novel VSC-HVDC link model for dynamic power system simulations

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This paper introduces a new RMS model of the VSC-HVDC link. The model is useful for assessing the steady-state and dynamic responses of large power systems with embedded back-to-back and point-to-point VSC-HVDC links. The VSC-HVDC model comprises two voltage source converters (VSC) linked by a DC cable. Each VSC is modelled as an ideal phase-shifting transformer whose primary and secondary windings correspond, in a notional sense, to the AC and DC buses of the VSC. The magnitude and phase angle of the ideal phase-shifting transformer represent the amplitude modulation ratio and the phase shift that exists in a PWM converter to enable either generation or absorption of reactive power purely by electronic processing of the voltage and current waveforms within the VSC. The mathematical model is formulated in such a way that the back-to-back VSC-HVDC model is realized by simply setting the DC cable resistance to zero in the point-to-point VSC-HVDC model. The Newton–Raphson method is used to solve the nonlinear algebraic and discretised differential equations arising from the VSC-HVDC, synchronous generators and the power grid, in a unified frame-of-reference for efficient, iterative solutions at each time step. The dynamic response of the VSC-HVDC model is assessed thoroughly; it is validated against the response of a detailed EMT-type model using Simulink\textsuperscript{®}. The solution of a relatively large power system shows the ability of the new dynamic model to carry out large-scale power system simulations with high efficiency.

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1. Introduction

Continuous increases in electrical energy consumption have encouraged a great deal of technological development in the electrical power industry. In particular, the development of new equipment for power transmission that enables a more flexible power grid aimed at achieving higher throughputs, enhancing system stability and reducing transmission power losses, has been high on the agenda [1,2]. The VSC-HVDC link is the latest equipment developed in the arena of high-voltage, high-power electronics and its intended function is to transport electrical power in DC form, as well as to enable the asynchronous interconnection of otherwise independent AC systems [3], and to provide independent reactive power support. The technology employs Insulated Gate Bipolar Transistors (IGBTs), driven by pulse width modulation (PWM) control. This valve switching control permits to regulate dynamically, in an independent manner, the reactive power at either terminal of the AC system and the power flow through the DC link [4].

The VSC-HVDC model put forward in this paper comprises two VSC models linked by a cable on its DC sides. In turn, each VSC model is made up of an ideal phase-shifting transformer which synthesises the phase-shifting and scaling nature of the PWM control. The ideal phase shifter is taken to be the interface between the AC and DC circuits of the VSC. The model makes provisions for the representation of conduction losses and switching losses. Since both converters are capable of independently controlling the reactive power exchanged with the AC power grid at their respective AC nodes, the VSC-HVDC dynamic model uses two independent dynamic voltage regulators. Both control loops are aimed at providing the required reactive power support at their respective AC nodes to maintain pre-set voltages, by regulation of their amplitude modulation coefficients. Likewise the model correctly accounts for the dynamics of the DC link. This is carried out by using a control block that acts upon the DC current to adjust the DC voltage of the VSC-HVDC link.

It should be mentioned that in an early model of the VSC-HVDC system, the two VSC are emulated by idealised voltage sources [5–8]. Alternatively, the VSCs have also been represented by equivalent controlled current sources [9,10], where the currents to be injected into the AC grids are computed by the existing difference between the complex voltages of the VSC terminals and the AC
system nodes at which the VSC-HVDC is embedded. More recently, the concept of dynamic average modelling has caught the attention of the power system community since it allows the modelling of VSCs in a more detailed manner [11]. In the dynamic average modelling approach, the average value of the output voltage waveform is calculated at each switching interval, a value that changes dynamically depending on the value of the reference waveform. The VSC is represented by a three-phase controlled voltage source on its AC sides and as a controlled current source on its DC sides [12]. However, this approach may be time consuming when repetitive simulations studies are required, such as in power grid expansion planning and in operation planning. The solution time is always an important point to keep in mind and in this method, increasing time steps is always a temptation but caution needs to be exercised when using the dynamic averaging method since, as reported in [12], the use of large time steps may affect the accuracy of the results. It is worth mentioning that if harmonics or electromagnetic transients are the study subject, such a high level of modelling detail is necessary, where, for instance, the PWM control needs to be modelled explicitly to achieve meaningful results.

On the other hand, in large-scale power system applications, it looks attractive to represent each VSC as a controlled voltage source owing to its much reduced complexity. However, its internal variables may not be readily available. In contrast, the new model introduced here captures very well the key operational characteristics of the VSCs making up the HVDC link. This is done by using explicit state variables that encapsulate the actual performance of the AC and DC circuits for both, the steady-state and dynamic operating regimes. Furthermore, the new VSC-HVDC model possesses the four degrees of freedom found in actual VSC-HVDC installations, characterised by having simultaneous voltage support at its two AC terminals, DC voltage control at the inverter converter and regulated DC power at the rectifier converter.

The numerical implementation of the VSC-HVDC model is carried out using a unified framework which suitably combines the algebraic and discretised differential equations of the VSC-HVDC link model, the synchronous generators and the non-linear algebraic equations of the power grid. This iterative solution takes advantage of the Newton–Raphson (NR) method thus facilitating the efficient solution of the non-linear equations. The discretisation of the differential equations is carried out using the implicit trapezoidal rule of integration which has been proven to be numerically stable and accurate [13,14]. In this paper, special attention is paid to the new dynamic VSC-HVDC model, emphasising how the algebraic and discretised differential equations are assembled together in this framework.

2. VSC-HVDC model for dynamic analysis

2.1. Key physical characteristics

If two VSC stations are linked as shown in Fig. 1, a VSC-HVDC system is formed and termed point-to-point configuration. In this arrangement, electric power is taken from one point of the AC network, converted to DC in the rectifier station, transmitted through the DC link and then converted back to AC in the inverter station and injected into the receiving AC network. In addition to transport power in DC form, this combined system is also capable of supplying reactive power and providing independent dynamic voltage control at its two AC terminals. It is worth mentioning that by setting the cable resistance $R_{DC}$ to zero, the representation reduces to that of the so-called back-to-back VSC-HVDC configuration. Please refer to the Appendix A for the symbols used in all equations and figures.

2.2. VSC-HVDC steady-state model

Fig. 2 depicts the equivalent circuit of the VSC corresponding to the inverter station; a similar topology can be formulated for the rectifier station. Its steady-state representation relies on an ideal phase-shifting transformer with complex taps, a series impedance on its AC side as well as an equivalent variable shunt susceptance $B_{eq}$ and a shunt resistor on its DC side [15].

The series reactance $X_{II}$ represents the VSC’s interface magnetism whereas the series resistor $R_{II}$ is associated to the ohmic losses which are proportional to the AC terminal current squared. The shunt resistor (with a conductance value of $G_{eq}$) produces power loss to account for the switching action of the converter valves. This conductance is calculated according to rated conditions and ensures that the operating conditions on switching losses are represented by scaling the quadratic ratio of the actual terminal current $I_{l}$ to the nominal current $I_{nom}: G_{eq} = G_{II} (I_{l}/I_{nom})^{2}$. Note that the squaring of this ratio is to give the switching conductance term an overall power performance. The following assumptions are made in the model: (a) the complex voltage $V_{1} = k_{V} m_{dc} E_{DC} e^{j\phi}$ is the voltage relative to the system phase reference; (b) the tap magnitude $m_{dc}$ of the ideal phase-shifting transformer corresponds to the VSC’s amplitude modulation coefficient where the following relationship holds for a two-level, three-phase VSC: $k_{V} = \sqrt{3}\beta$; (c) the angle $\phi_{l}$ is the phase angle of voltage $V_{l}$; (d) $E_{DC}$ is the DC bus amplitude voltage which is a real scalar. Bearing this in mind, the nodal power flow equations for the series branch of the VSC representing the inverter station are derived from the nodal admittance matrix developed in Appendix B. After some arduous algebra, the active and reactive powers expressions for the powers injected at both ends of the VSC, nodes $vl$ and $0vl$, are arrived at:

$$P_{vl} = V_{vl}^{2} G_{II} - k_{V} m_{dc} V_{vl} E_{DC} \left[ G_{II} \cos (\theta_{vl} - \phi_{l}) + B_{II} \sin (\theta_{vl} - \phi_{l}) \right]$$

(1)

$$Q_{vl} = -V_{vl}^{2} B_{II} - k_{V} m_{dc} V_{vl} E_{DC} \left[ G_{II} \sin (\theta_{vl} - \phi_{l}) - B_{II} \cos (\theta_{vl} - \phi_{l}) \right]$$

(2)

$$P_{0vl} = k_{V}^{2} m_{dc}^{2} E_{DC} G_{II} - k_{V} m_{dc} V_{vl} E_{DC} \left[ G_{II} \cos (\phi_{l} - \theta_{vl}) + B_{II} \sin (\phi_{l} - \theta_{vl}) \right] + P_{eql}$$

(3)

$$Q_{0vl} = -k_{V}^{2} m_{dc}^{2} E_{DC} B_{II} - k_{V} m_{dc} V_{vl} E_{DC} \left[ G_{II} \sin (\phi_{l} - \theta_{vl}) - B_{II} \cos (\phi_{l} - \theta_{vl}) \right] + Q_{eql}$$

(4)
where,
\[ p_{sw1} = E^2_{DC1} C_{DC1} (I_{d1}/I_{nam})^2 \]  
(5)
\[ Q_{eqf} = -\frac{\Delta_1}{\Delta_2} E_{DC1} B_{eqf} \]  
(6)

Likewise, a similar set of equations may be obtained for the VSC corresponding to the rectifier station. To obtain the steady-state equilibrium point, the set of mismatch power flow equations that must be solved together with those arising from all the network’s nodes is:
\[ \Delta P_{OR} = -P_{OR} - P_{OR,\text{load}} - P_{OR,\text{cal}} = 0 \]  
(7)
\[ \Delta Q_{OR} = -Q_{OR} - Q_{OR,\text{load}} - Q_{OR,\text{cal}} = 0 \]  
(8)
\[ \Delta P_{in} = -P_{in} - P_{in,\text{load}} - P_{in,\text{cal}} = 0 \]  
(9)
\[ \Delta Q_{in} = -Q_{in} - Q_{in,\text{load}} - Q_{in,\text{cal}} = 0 \]  
(10)
\[ \Delta E_{OR} = E_{DCR}^2 - E_{DRC} E_{DRO} - P_{sch} R_{DC} = 0 \]  
(11)
\[ \Delta P_{OR} = -P_{OR} - P_{OR,\text{cal}} = 0 \]  
(12)
\[ \Delta P_{in} = P_{sch} - (E_{DRO} - E_{DC})^2 R_{DC} - P_{in,\text{cal}} = 0 \]  
(13)
\[ \Delta Q_{OR} = -Q_{OR} - Q_{OR,\text{cal}} = 0 \]  
(14)
\[ \Delta Q_{in} = -Q_{in} - Q_{in,\text{cal}} = 0 \]  
(15)

where, in this particular case, \( P_{OR,\text{cal}}, Q_{OR,\text{cal}}, P_{in,\text{cal}} \) and \( Q_{in,\text{cal}} \), stand for the powers flowing from bus \( v \) to \( k \) and \( v \) to \( m \), respectively. They are given by
\[ P_{OR,\text{cal}} = V_{vR}^2 B_{R} + V_{vR} V_{k} \left[ G_{Rk} \cos (\theta_{vR} - \theta_{k}) + B_{Rk} \sin (\theta_{vR} - \theta_{k}) \right] \]  
(16)
\[ Q_{OR,\text{cal}} = -V_{vR}^2 B_{R} + V_{vR} V_{k} \left[ G_{Rk} \sin (\theta_{vR} - \theta_{k}) - B_{Rk} \cos (\theta_{vR} - \theta_{k}) \right] \]  
(17)

Similar equations may be obtained for the power flowing from bus \( t \) to \( m \) by simply exchanging subscripts in (16) and (17). It should be remarked that (12) ensures that the power flowing leaving the rectifier station be kept at the scheduled value \( P_{sch} \). Given that the inverter station is chosen to keep the DC link voltage at a constant value \( E_{DC} \) then Eq. (11) allows the computation of the DC voltage in the rectifier’s side, \( E_{DRO} \). Since the objective is to regulate the voltage magnitude at both AC sides of the VSC-HVDC while keeping the DC voltage fixed, \( V_{DC1}, V_{DC2} \) and \( E_{DRO} \) are not part of the set of state variables that need to be computed. Thus, \( \theta_{vR}, m_{Rk}, \theta_{k}, m_{k}, E_{DRC}, \phi_{R}, \phi_{K}, B_{Rk} \), and \( B_{eqf} \), constitute the set of state variables that must be calculated by solving (7)–(15) with the rest of the equations arising from the network. To guarantee that each VSC operates within feasible operating limits, a limit checking of the modulation ratio and terminal current must take place, that is, \( m \leq 1 \) and \( I \leq \text{I_{nam}} \). Furthermore, to enable good starting conditions, the NR algorithm is initialised as follows: the amplitude modulation ratios, \( m_{Rk} \) and \( m_{k} \), and the angles, \( \phi_{R} \) and \( \phi_{K} \), are set at 1 and 0, respectively.

### 2.3. VSC-HVDC dynamic model

The VSC capacitor’s dynamics plays an important role in the behaviour of the HVDC link when subjected to voltage and power variations coming from the external AC network. On the other hand, in this VSC application, DC voltage control is a target in order to pursue a stable operation of the DC link. The inverter converter is the one that takes on such a task, where the following relationship holds at its DC terminals: \( i_c = -I_{DRC} - I_{DCI} \), being \( I_{DRC} \) and \( I_{DCI} \) the currents injected at rectifier’s DC bus and at inverter’s DC bus, respectively. Substituting this current relationship into the expression that allows calculating the capacitor’s current, \( i_c = C_{DC} \frac{dE_{DC}}{dt} \), we get the differential equation with which the DC voltage dynamics are represented.

\[ \frac{dE_{DCI}}{dt} = -\frac{I_{DRC} - I_{DCI}}{C_{DC}} \]  
(18)
\[ I_{DRC} = \frac{P_{OR} - P_{OR,\text{cal}}}{E_{DCR}} \]  
(19)

The value of \( C_{DC} \) is estimated from the amount of energy stored in the capacitor: \( W_{C} = \frac{1}{2} C_{DC} E_{DC}^2 \). The electrostatic energy (capacitive) in the DC capacitor can be associated with an equivalent inertia constant \( H_{c} [s] \) as \( W_{C} = H_{c} \text{S_{IC}} \), where \( \text{S_{IC}} \) would correspond to the rated apparent power of the VSC. This time constant is small and it may be taken to be \( H_{c} \approx 5 \text{ ms} \) [16]. Hence, the per-unit value of the capacitor would be \( C_{DC} = 2 \text{S_{IC}} H_{c} / E_{DC}^2 \).

Arguably, the current balance shown in (18) is akin to the power balance inside the HVDC link for steady-state operating conditions when the derivative term becomes zero. Thus, whenever the current/power balance is disturbed, voltage variations will appear in the DC link. As shown in Fig. 3, the dynamic control of the HVDC’s DC voltage is carried out by using the DC current entering the inverter converter, \( I_{DCI} \), as the control variable. The error between the actual voltage \( E_{DCI} \) and \( E_{DCI,\text{nom}} \) is used by a PI controller, with gains \( K_{pedc} \) and \( K_{I pedc} \), to obtain new values of DC current \( I_{DCI} \).

The differential and algebraic equations arising from the DC voltage dynamic controller are

\[ \frac{dI_{DCI,aux}}{dt} = K_{pedc} (P_{sch} + P_{OR}) \]  
(20)
\[ I_{DCI} = K_{pedc} (E_{DCI} - E_{DCI,\text{nom}}) + I_{DCI,aux} \]  
(21)

Simultaneously, the rectifier converter must ensure that the active power leaving this station be kept at the scheduled value \( P_{sch} \). From Fig. 1, it can be inferred that the power entering the inverter station is \( P_{sch} \) minus the power loss incurred by the DC cable resistor. If the inverter station is selected to perform the control of the DC link voltage \( E_{DCI} \) then the DC voltage at the rectifier’s side, \( E_{DCR} \), can be computed at any time by applying Kirchhoff’s voltage law in the DC circuit, as follows:

\[ E_{DCR} = E_{DC1} - R_{DC} I_{DCR} \]  
(22)

The angular aperture of the phase-shifting angle of the rectifier \( \phi_{R} \) and the voltage angle \( \theta_{R} \) is related to the power exchange occurring at any time between the network and the rectifier’s DC bus. Hence, the angular difference \( \gamma_{R} = \theta_{R} - \phi_{R} \) is also a key parameter that requires proper regulation with the aim to achieve the scheduled active power transfer \( P_{sch} \) from the rectifier station towards the inverter station. Then, the pursued power balance on the DC side will now be given by the following expression: \( P_{OR} + P_{sch} = 0 \), as shown in Fig. 4.

The equations that allow the assessment of the dynamic behaviour for the scheduled power controller are

\[ \frac{dI_{DCI}}{dt} = K_{pedc} (P_{sch} + P_{OR}) \]  
(23)
The AC voltage dynamic control of the VSC-HVDC calls for two control loops, as shown in the first-order control blocks of Fig. 5. The modulation indices \( m_{al} \) and \( m_{ar} \) are responsible for either, controlling the voltage magnitudes at the AC sides of the rectifier and inverter stations of at the scheduled values, \( V_{R0} \) and \( V_{I0} \), or to exert the fixed reactive power setpoint: \( Q_{ref} \) and \( Q_{R} \). The controls are designed in such a way that the modulation indices \( m_{al} \) and \( m_{ar} \) are readjusted at every time step according to the voltage or reactive power commands.

The differential equations representing the dynamics of the modulation indices when voltage control is selected are

\[
\frac{d(m_{al})}{dt} = \frac{k_{mal}(V_{R0} - V_{al}) - m_{al}}{t_{mal}} \quad (25)
\]

\[
\frac{d(m_{ar})}{dt} = \frac{k_{mar}(V_{I0} - V_{ar}) - m_{ar}}{t_{mar}} \quad (26)
\]

3. Dynamic frame of reference

In this paper the interest is in assessing the effectiveness of the new VSC-HVDC model to regulate voltage magnitude at either terminal of the AC system, following a change in the power network such as a step change in system load or the tripping of a transmission line or transformer. Hence, the solution method presented in [13], is selected to implement the VSC-HVDC model developed in Section II. This approach combines the set of algebraic equations (27) representing the power network with the system of differential equations (28) describing the dynamic behaviour of the synchronous generators and their controls, to obtain the solution as a function of time in a unified frame of reference. It uses the implicit trapezoidal method (see Appendix C) which is known to be numerically stable, preserving a reasonable accuracy [13,14],

\[ 0 = f(X, Y) \]

\[ \dot{y} = g(X, Y, t) \]

where \( X \) and \( Y \) are vectors of variables that are computed at discrete points in time.

These equations are efficiently solved using the NR method. In this case, the conventional power flow jacobian matrix, \( J \), is enlarged to accommodate the partial derivatives that arise from the discretised differential equations and its control variables. The NR method provides an accurate solution to the set of equations given by \( F(Z) = 0 \), by solving for \( \Delta Z \) in the linearised problem \( j\Delta Z = -F(Z) \), in a repetitive fashion. In this case \( Z \) is a vector that contains the network’s state variables and the state variables pertaining to the synchronous generators and their controls or, indeed, any other control device such as the VSC-HVDC link. In an expanded form,

\[
\begin{bmatrix}
\Delta P \\
\Delta Q \\
\Delta F(y)
\end{bmatrix} = -
\begin{bmatrix}
\frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial y} \\
\frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial y} \\
\frac{\partial F(y)}{\partial \theta} & \frac{\partial F(y)}{\partial V} & \frac{\partial F(y)}{\partial y}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta \\
\Delta V \\
\Delta y
\end{bmatrix}
\]

where \( \Delta P \) and \( \Delta Q \) are the active and the reactive power mismatch vectors, respectively; \( F(y) \) is a vector that contains the discretised differential equations of each machine or controlling device; \( \Delta \theta \), \( \Delta V \) and \( \Delta y \) represent the vectors of incremental changes in nodal voltage angles and magnitudes, as well as the state variables arising from each differential equation. The NR method starts from an initial guess for \( Z_0 \) and updates the solution at each iteration \( i \), i.e., \( Z_i = Z_0 + \Delta Z_i \) until a predefined tolerance is fulfilled. In this unified solution, all of the state variables are adjusted simultaneously in order to compute the new equilibrium point of the power system at every time step.

3.1. Discretisation and linearisation of the VSC-HVDC equations for dynamic simulations

To enable a suitable representation in this unified frame of reference, the VSC-HVDC differential equations are discretised and expressed in the form of a mismatch equation in the same form as that of the network’s active and reactive power mismatch equations:

\[
F_{E_{DCI}} = E_{DCI,t-\Delta t} + \frac{\Delta t}{2} E_{DCI,t-\Delta t} - \left( E_{DCI,t} - \frac{\Delta t}{2} E_{DCI,t} \right) = 0 \quad (30)
\]

\[
F_{I_{DCaux}} = I_{DCaux,t-\Delta t} + \frac{\Delta t}{2} I_{DCaux,t-\Delta t} - \left( I_{DCaux,t} - \frac{\Delta t}{2} I_{DCaux,t} \right) = 0 \quad (31)
\]

\[
F_{\gamma_{raux}} = \gamma_{raux,t-\Delta t} + \frac{\Delta t}{2} \gamma_{raux,t-\Delta t} - \left( \gamma_{raux,t} - \frac{\Delta t}{2} \gamma_{raux,t} \right) = 0 \quad (32)
\]

\[
F_{dm_{al}} = dm_{al,t-\Delta t} + \frac{\Delta t}{2} dm_{al,t-\Delta t} - \left( dm_{al,t} - \frac{\Delta t}{2} dm_{al,t} \right) = 0 \quad (33)
\]

\[
F_{dm_{ar}} = dm_{ar,t-\Delta t} + \frac{\Delta t}{2} dm_{ar,t-\Delta t} - \left( dm_{ar,t} - \frac{\Delta t}{2} dm_{ar,t} \right) = 0 \quad (34)
\]

where,

\[
\dot{E}_{DCI} = C_{DC} \left( -I_{DCR,t} - I_{DCI,t} \right)
\]
$E_{\text{DCI},t-\Delta t} = C_{\text{DC}} \left( I_{\text{DCI},t-\Delta t} - I_{\text{DCI},t-\Delta t} \right)$ \hspace{1cm} (36)

$i_{\text{DCI},t,t} = K_{\text{edc}} \left( E_{\text{DCI},t} - E_{\text{DCInom}} \right)$ \hspace{1cm} (37)

$i_{\text{DCIaux},t,t} = K_{\text{edc}} \left( E_{\text{DCI},t} - E_{\text{DCInom}} \right)$ \hspace{1cm} (38)

$\dot{y}_{\text{raux},t,t} = K_{\text{pdc}} \left( P_{\text{sch}} + P_{\text{dv,rt,t}} \right)$ \hspace{1cm} (39)

$\dot{y}_{\text{raux},t,t} = K_{\text{pdc}} \left( P_{\text{sch}} + P_{\text{dv,rt,t}} \right)$ \hspace{1cm} (40)

$\frac{dm_{\text{R},t}}{dt} = T_{\text{mR}}^{-1} \left[ K_{\text{mR}} \left( V_{\text{iR},t} - V_{\text{R},t} \right) - dm_{\text{R},t} \right]$ \hspace{1cm} (41)

$\frac{dm_{\text{I},t}}{dt} = T_{\text{mI}}^{-1} \left[ K_{\text{mI}} \left( V_{\text{iI},t} - V_{\text{I},t} \right) - dm_{\text{I},t} \right]$ \hspace{1cm} (42)

$\frac{dm_{\text{I},t}}{dt} = T_{\text{mI}}^{-1} \left[ K_{\text{mI}} \left( V_{\text{iI},t} - V_{\text{I},t} \right) - dm_{\text{I},t} \right]$ \hspace{1cm} (43)

$\frac{dm_{\text{R},t}}{dt} = T_{\text{mR}}^{-1} \left[ K_{\text{mR}} \left( V_{\text{iR},t} - V_{\text{R},t} \right) - dm_{\text{R},t} \right]$ \hspace{1cm} (44)

The Eqs. (30)–(44) govern the dynamic behaviour of the VSC-HVDC model. The first two equations capture the DC voltage and current performance of the DC link when the energy balance is perturbed owing to a disturbance in the AC network. Likewise, the equation involving the angular aperture $\gamma_{\text{R}}$ (32) deals with the power unbalance present in the DC link. Also, the Eqs. (33) and (34) enable the computation of the new values of the modulation indices with which the target AC voltages are acquired for the actual AC network's operating conditions. In order to link the VSC-HVDC's control variables with the grid's state variables at nodes $\text{vR}$ and $\text{vI}$, the algebraic power mismatch Eqs. (7)–(10) must be used. However, to complete the model for dynamic simulation purposes, two more algebraic equations are needed. One for calculating the voltage $E_{\text{DCR}}$ at the rectifier station's DC bus (45) and another to enable the HVDC to achieve the power balance at the inverter's DC bus (46).

$\Delta E_{\text{QR}} = E_{\text{DCR}} - E_{\text{DCI}} + R_{\text{DCI}} I_{\text{DCI}}$ \hspace{1cm} (45)

$\Delta P_{\text{RI}} = E_{\text{DCR}} I_{\text{DCI}} - P_{\text{RI}}$ \hspace{1cm} (46)

Eqs. (7)–(10), (45)–(46) and (30)–(34) constitute the set of mismatch equations that must be assembled together with the equations of the whole network, synchronous generators and their corresponding controllers. The linearised form of the VSC-HVDC mathematical model is given by,

$\Delta F = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \Delta z$ \hspace{1cm} (47)

$\Delta F = \begin{bmatrix} \Delta P_{\text{RI}} & \Delta Q_{\text{RI}} & \Delta P_{\text{EI}} & \Delta Q_{\text{EI}} & \Delta E_{\text{QR}} & \Delta P_{\text{RI}} & F_{\text{DCI}} & F_{\text{DCIaux}} & F_{\gamma_{\text{Raux}}} & F_{dm_{\text{RI}}} & F_{dm_{\text{EI}}} \end{bmatrix}^T$

$\Delta z = \begin{bmatrix} \Delta \theta_{\text{RI}} & \Delta V_{\text{RI}} & \Delta \theta_{\text{EI}} & \Delta V_{\text{EI}} & \Delta E_{\text{DCR}} & \Delta \phi_{t} & \Delta E_{\text{DCI}} & \Delta I_{\text{DCIaux}} & \Delta \gamma_{\text{Raux}} & \Delta dm_{\text{RI}} & \Delta dm_{\text{EI}} \end{bmatrix}^T$

where $J_{11}$ comprises the first-order partial derivatives of the power mismatch equations and inner VSC-HVDC's mismatch equations with respect to the network's and VSC-HVDC's state variables. Likewise, $J_{12}$ contains the first order partial derivatives arising from the algebraic mismatch equations with respect to the control variables of the VSC-HVDC link. The matrix $J_{21}$ consists of partial derivatives of the VSC-HVDC's discretised differential equations with respect to the AC voltages and angles, the phase-shifting angle $\phi_{t}$ and the DC voltage $E_{\text{DCR}}$. Lastly, $J_{22}$ is a matrix that accommodates the first-order partial derivatives of the VSC-HVDC's discretised differential equations with respect to their own control variables.

The steady-state conditions that are employed to start the dynamic simulation are calculated through the conventional Newton–Raphson power flow algorithm including the VSC-HVDC link steady-state model, as discussed in Section 2.2.2. Such a solution will provide adequate starting conditions to ensure reliable dynamic simulations.
4. Study Cases

4.1. Validation of the new VSC-HVDC model

The prowess of the new VSC-HVDC link model is demonstrated by carrying out a comparison against the widely-used EMT-type simulation software Simulink®. It should be mentioned that both types of simulation tools enable dynamic assessments of electrical power networks but they take a fundamentally different approach. Simulink® represents every component of the power grid by means of RLC circuits and their corresponding differential equations require discretisation at rather small time steps, in the order of micro-seconds, to ensure a stable numerical solution. Conversely, the solution of the RMS-type model introduced in this paper requires only one phase of the network (positive sequence), using fundamental-frequency phasors of voltages and currents as opposed to the three-phase representation along with instantaneous waveforms of voltages and currents used in an EMT tool such as Simulink®.

The VSC-HVDC model comparison is carried out using a rather simple power system comprising two independent AC networks (2000 MVA, 230 kV, 50 Hz) which are interconnected through a VSC-HVDC link (200 MVA, ±100 kV DC) with a DC cable length of 75 km, as shown in Fig. 6. Both converter stations comprise each a step-down transformer, AC filters, converter reactor, DC capacitors and DC filters, where the changes of the transformers' tap are not simulated. The model of the power system including the VSC-HVDC link together with its parameters can be found in the section of 'demos' in Simulink® as: VSC-Based HVDC Transmission System (Detailed Model), whereas the parameters of the new VSC-HVDC model are shown in Appendix D. To ensure a reliable numerical solution, the EMT-type simulation package discretises the power system and the control system with a sample time of 7.406 μs and 74.06 μs, respectively, whereas for the developed RMS-type model, an integration step of 1 ms is used. All the results shown in p.u. values are based on the HVDC station’s rating.

Initially the rectifier station is set to control the active power transmission at \( P_{\text{rect}} = 200 \text{ MW} \) (1 p.u.), the inverter is responsible for controlling the DC voltage at \( E_{\text{DC,lim}} = 200 \text{ kV} \) (1 p.u.). The rectifier and inverter stations are set to comply with a fixed reactive power command of 0 p.u. and −0.1 p.u., respectively. In order to reach the steady-state equilibrium point in Simulink®, the simulation is run up to \( t = 1 \text{ s} \). At this point, the active power transmission is reduced from 200 MW to 100 MW, that is, a −50% step is applied to the reference scheduled DC power. Furthermore, at \( t = 3 \text{ s} \), a step change of −5% is applied to the reference DC voltage of the inverter, i.e., the DC voltage is decreased from 1.0 p.u. to 0.95 p.u.

The DC voltages at the converters' DC terminals are shown in Fig. 7 corresponding to cases where step changes in the reference DC power and DC voltage are applied. As expected, some differences can be seen from the results obtained using both solution techniques. The dynamic performance of the DC voltages of the RMS-type model follows well the dynamic pattern obtained by the switching-based HVDC model simulated in Simulink®. A very considerable difference exists between the two approaches at the start of the simulation (0.5 s of the simulation), a fact that can be explained by the very different manner in which both power system simulations are initialised; our proposed VSC-HVDC system uses an accurate starting condition furnished by a power flow solution whereas the Simulink® model starts from its customary zero initial condition, i.e., the currents and voltages of the inductors and capacitors, respectively, are set to zero at \( t = 0 \text{ s} \).

Similar conclusions can be drawn when analysing the dynamic response of the DC power following the application of the step changes in DC power and DC voltage, as shown in Fig. 8. As for the change in the DC power reference, it can be seen that the power stabilises in no more than 0.5 s; this shows the rather quick response and robustness afforded by the dynamic controls of the VSC-HVDC link even in the event of a drastic change in the transmitted DC power. On the other hand, the step change in the DC voltage reference causes momentary power flow oscillations in the DC link which are also damped out quite rapidly.

The dynamic behaviour of the HVDC's modulation indices are depicted in Fig. 9. The negative step change in the DC power reference yields a very noticeable variation in the modulation indices; the dynamic performance of the modulation indices as calculated by both Simulink® and the proposed HVDC model, follow the same trend although an exact match was not expected. After the first disturbance, a steady-state error of 0.87% and 1.67% is obtained for the modulation indices of the rectifier and inverter, respectively. Similarly, once the oscillations due to the step change in the reference DC voltage have been damped, the differences in the modulation indices stand at 0.07% and 1.16%, respectively. These relatively small variations may be explained by the very different modelling and solution approaches used by the two quite different simulation techniques used for the comparison, the initial steady-state values of the converters' indices and most importantly due to

![Fig. 6. Test system used to validate the proposed VSC-HVDC model.](image)

![Fig. 7. DC voltage performance for the proposed and Simulink VSC-HVDC model.](image)

![Fig. 8. DC power performance for the proposed and Simulink VSC-HVDC model.](image)
the DC voltage behaviour as this has a strong impact on the performance of the modulation indices. More importantly, the results furnished by the two software simulations follow the same trend.

For the sake of comparison, Table 1 shows the VSC-HVDC results as obtained by the new model and the Simulink® model at different points in time of simulation. Table 1 also shows the computing times required to simulate the test system using both the RMS-type VSC-HVDC model and the EMT-type simulation tool Simulink®. The new model being approximately nine times faster than the EMT simulation. The significant computational time saving without jeopardising the accuracy of the results makes the developed VSC-HVDC link model a suitable option for large-scale power system simulations, specifically in studies that require longer simulation times such as those involving synchronous generators’ frequency variations and long-term voltage stability issues.

4.2. New England test system with embedded VSC-HVDC link

The New England test system [17] is modified, as shown in Fig. 10, to incorporate the model of a VSC-HVDC with the parameters shown in Appendix D. The transmission line connecting nodes 4 and 14 is replaced by a VSC-HVDC link. The DC cable resistance is assumed to be 0.24% on the VSCs’ base: $S_{\text{nom}} = 300$ MVAR, resulting in the same resistance value as that of the replaced transmission line for the system’s base: 0.08%. The rectifier and inverter stations, VSCs and VSCs, exert voltage control at their respective AC terminals at $V_{\text{ref}} = 1.01$ p.u and $V_{\text{ref}} = 1.03$ p.u, respectively. For the steady-state conditions, the high-voltage side of the LTC transformers, which correspond to nodes 4 and 14, are held fixed at the same voltage level as those for the converters’ terminals, $V_{\text{ref}}$ and $V_{\text{ref}}$; under these conditions, the LTC’s taps are computed through the steady-state power flow algorithm; their values are kept constant during the dynamic solution. In addition to providing reactive power control, the HVDC link performs active power regulation at the rectifier station’s DC bus at $P_{\text{ach}} = 100$ MW, which implies that the active power is drawn from node 4 and injected to node 14, as depicted in Fig. 10. The active and reactive powers presented in the analysis for the steady-state and dynamic operating regimes are given at the high-voltage side of each LTC transformer. All results shown in p.u. values are based on the HVDC station’s ratings.

During steady state, the rectifier station is delivering 153.359 MVAR to the network so as to uphold its target voltage magnitude with a modulation ratio of 0.8282, whereas the inverter station operates with a modulation ratio of 0.8423, injecting 27.306 MVAR to the grid. In the case of the active power flowing through the HVDC system, the power entering the rectifier station stands at 101.159 MW and the power leaving the inverter station takes a value of 99.641 MW. It is clear that the difference between these two powers is the total power loss incurred by the HVDC system including that produced by the DC link cable. Taking as a reference the nominal apparent power for each converter $S_{\text{nom}}$, the total power losses stand at 0.504 of which 0.386 corresponds to the rectifier station and 0.112% to the inverter station whilst the power loss produced by Joule’s effect in the DC cable stands at 0.006, recalling that its magnitude is dependent on the length of the DC transmission line. Table 2 shows the main VSC-HVDC results as given by the steady-state power flow solution which serves the purpose of initialising the dynamic simulation.

Hence, using values from the steady-state power flow solution, it is easy to proceed with the calculation of the initial values for the control variables taking part in the dynamics of the VSC-HVDC link; these are employed to initialise the dynamic simulation of the test network, when subjected to the disconnection of the transmission lines linking nodes 25-2, 2-3 and 3-4, at $t = 0.1$ s.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Proposed model</th>
<th></th>
<th>Simulink® model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{\text{OCR}}$</td>
<td>$E_{\text{CFC}}$</td>
<td>$m_{\text{ref}}$</td>
<td>$m_{\text{ref}}$</td>
</tr>
<tr>
<td>$t = 1$</td>
<td>1.0105</td>
<td>1.0000</td>
<td>0.8553</td>
<td>0.8296</td>
</tr>
<tr>
<td>$t = 3$</td>
<td>1.0053</td>
<td>1.0000</td>
<td>0.8389</td>
<td>0.8172</td>
</tr>
<tr>
<td>$t = 5$</td>
<td>0.9556</td>
<td>0.9500</td>
<td>0.9329</td>
<td>0.8711</td>
</tr>
<tr>
<td></td>
<td>Computing time: 19.76 s</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2
Computed VSC-HVDC variables by the power flow solution.

<table>
<thead>
<tr>
<th></th>
<th>( Q_{\text{per}} )</th>
<th>( E_{\text{DC}} )</th>
<th>( m_x )</th>
<th>( \phi )</th>
<th>( B_{\text{eq}} )</th>
<th>LTC's tap</th>
<th>( P_{\text{trans}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VSC</td>
<td>153.359 MVAR</td>
<td>1.0002 p.u.</td>
<td>0.8282</td>
<td>-2.6655(^\circ)</td>
<td>0.5182 p.u.</td>
<td>1.0252</td>
<td>1.1595 MW</td>
</tr>
<tr>
<td>VSC</td>
<td>27.306 MVAR</td>
<td>1.0000 p.u.</td>
<td>0.8423</td>
<td>6.8416(^\circ)</td>
<td>0.0918 p.u.</td>
<td>1.0044</td>
<td>0.3383 MW</td>
</tr>
</tbody>
</table>

Fig. 12. Dynamic behaviour of the converters’ modulation indices.

Fig. 13. Reactive power generated by both converters making up the HVDC link.

Fig. 14. DC current behaviour for the rectifier and inverter.

Fig. 15. DC voltage behaviour for the rectifier and inverter.

Fig. 16. VSC-HVDC's AC active power and DC-power transfer behaviour.

Fig. 11 shows the voltage magnitudes at various nodes following a change in the network’s topology. During the transient period, the target voltage set point is achieved very quickly by the action of the AC-bus voltage controllers that regulate the converters’ modulation indices \( m_{\text{af}} \) and \( m_{\text{al}} \), as shown in Fig. 12. The rather prompt action of both controllers leads to very rapid reactive power injection at both converters AC terminals, as can be seen in Fig. 13, resulting in the very effective damping of the voltage oscillations and enabling a smooth voltage recovery throughout the grid.

As soon as the disturbance takes place in the AC system, the energy balance in the DC link is broken; the voltage sags that take place at both converters AC terminals reduce the active power being transferred through the DC link; the current \( -I_{\text{DCR}} \) that flows from the rectifier station VSC\(_R\) towards the inverter station VSC\(_I\) drops abruptly from 0.166 p.u to 0.146 p.u, as illustrated by the blue line in Fig. 14. A momentary mismatch between both converters DC currents is then produced because the DC current cannot be instantly re-established due to the time constants involved in the current controller of the inverter station; as a result, DC voltage deviations take place, as shown in Fig. 15, reaching a minimum value of 0.978 p.u during the transient event. Nevertheless, once this controller starts responding to the DC voltage variations, the current \( I_{\text{DCR}} \), depicted by the green line in Fig. 14, starts tracing the DC current of the rectifier \( I_{\text{DCR}} \) to compensate for the voltage drop, enabling a speedy recovery of the DC link voltage. It should be remarked that in view of the fact that the cable resistance is relatively small, so is the voltage drop along the DC transmission line, resulting in quite similar magnitudes and, of course, dynamic behaviours of the voltages at both DC buses, \( E_{\text{DCR}} \) and \( E_{\text{CDI}} \).

The simulation results for the active power and DC-power transfer following the disconnection of the transmission lines close to the HVDC link are illustrated in Fig. 16. The blue line represents the active power entering the high-voltage side of the load-tap changer transformer coupled to the rectifier station whereas the green line characterizes the active power performance at the inverter’s LTC’s high-voltage side. The power difference represents the power losses incurred by the VSC-HVDC link, including those produced by the DC cable. The DC-power transfer \( P_{\text{DC}} \) consisting of the product of voltage \( E_{\text{DCR}} \) and current \( -I_{\text{DCR}} \) is also shown in the same graph. Since the voltage and current controls have been shown to operate efficiently, as illustrated in Fig. 14 and Fig. 15, then a fast power recovery is achieved in spite of the severe disturbance occurred in the network. Given that the power flowing from the rectifier towards the inverter station has been brought back to its initial target power transfer of 0.333 p.u, the deviation of the power angle \( \gamma_{\text{p}} \) suffers a mere marginal increase, as can be seen in Fig. 17, only to agree with the new reached steady-state conditions where different currents and, therefore, active power losses are produced. Fig. 17 also shows the behaviour of the phase-shifting angles of
the rectifier and inverter converters, $\phi_R$ and $\phi_I$. As expected, these angles follow the same pattern as those obtained by the network’s voltage angles $\theta$, at the nodes where the VSC-HVDC system is connected.

5. Conclusions

A new VSC-HVDC model for RMS dynamic simulations of large-scale power systems has been introduced in this paper. This is an all-encompassing model that facilitates the back-to-back and point-to-point representation of the VSC-HVDC by simply modifying the DC cable resistance value. The model possesses four degrees of freedom, a characteristic that conforms to actual VSC-HVDC links, i.e., it exerts simultaneous voltage control on its two AC terminals and at its DC bus and transmitted power through the DC link.

The model solution is carried out using the NR method which solves simultaneously the algebraic and differential equations at each time step. The point-to-point VSC-HVDC model comprises two series-connected VSC stations and a DC cable. Each VSC model uses an ideal phase-shifting transformer as its core element. The conduction losses and the switching losses of the HVDC converters are well captured in the model. Furthermore, the VSC-HVDC dynamic model is fitted with independent controllers for the AC and DC circuits to represent the quite distinct dynamic performances of the two control circuits.

The point-to-point VSC-HVDC model introduced in this paper has been validated using the EMT simulation tool Simulink®, where the results obtained from the two fundamentally different approaches agreed quite well with each other. Nevertheless, it was shown that the greater level of detail needed in an EMT solution comes with an onerous price-tag to pay in terms of a very considerable computational time compared to the computing time incurred by the new RMS-type VSC-HVDC model.

The performance of the new VSC-HVDC model was tested in a larger network which is widely used in academic circles, comprising 39 nodes. The VSC-HVDC link model performed well in terms of its modelling flexibility and in attaining the set control targets. It was shown that the DC current controller and the rectifier’s angular aperture controller operate efficiently to stabilize both the DC voltage and the DC power, respectively. Likewise, the AC voltage control was quickly achieved due to the proper action of the controllers acting upon the modulation indices of the converters.

Appendix A.

$E_{DC}$: DC voltage. $I_{DC}$: DC current. $C_{DC}$: DC capacitance. $R_{DC}$: DC transmission line resistance. $I_{nom}$: VSC’s nominal current. $S_{nom}$: VSC’s nominal power. $P_{sch}$: Scheduled active power. $m_p$: VSC’s modulation index. $\phi$: VSC’s phase-shifting angle. $k_2 = \sqrt{3}8$: Constant for a two-level, three-phase VSC. $C_{0}$: Shunt resistor which accounts for the switching losses of the VSC. $B_{eq}$: Equivalent variable shunt susceptance. $Y_1$: VSC’s series admittance with conductive losses and interface magnetics. $V$: Complex nodal voltage. $I$: Complex current injection. $S$: Complex nodal power injection. $k_{pdc}$, $k_{ipdc}$: Proportional and integral gains for the DC voltage control. $k_{pupdc}$, $k_{ipupdc}$: Proportional and integral gains for the DC power control. $k_{ma}$, $k_{ima}$: Proportional gain and time constant for the modulation index control. Subscripts $R$ and $I$ stand for rectifier and inverter, respectively.

Appendix B.

In connection with Fig. 2, the voltage and current relationships in the ideal phase-shifting transformer are:

$$\frac{V_1}{E_{DCI}} = \frac{k_2 m_{d1} \angle \phi_I}{I} \quad \text{and} \quad \frac{k_2 m_{d1} \angle - \phi_I}{I} = \frac{I_2}{I_1} \tag{A.1}$$

The current through the impedance connected between $vl$ and 1 is:

$$I_1 = Y_1 (V_{d1} - V_{p1}) = Y_1 V_{d1} - k_2 m_{d1} \angle \phi_I Y_1 E_{DCI} = I_{dl} \tag{A.2}$$

where $Y_1 = (R_1 + jX_1)^{-1}$.

At node 0vl, the following relationship holds,

$$I_{0vl} = -I_2 + G_{swl} E_{DCI} = -k_2 m_{d1} \angle - \phi_I Y_1 V_{d1} + k_2^2 m_{d1}^2 Y_1 E_{DCI} + jB_{eq} k_2^2 m_{d1} E_{DCI} + G_{swl} E_{DCI} \tag{A.3}$$

Rearranging Eqs. (A.2) and (A.3) yields:

$$\begin{bmatrix} I_2 \\ I_{0vl} \end{bmatrix} = \begin{bmatrix} 1 \\ -k_2 m_{d1} \angle \phi_I Y_1 \\ -k_2 m_{d1} \angle - \phi_I Y_1 \end{bmatrix} \begin{bmatrix} Y_1 \\ -k_2 m_{d1}^2 (Y_1 + jB_{eq}) + G_{swl} \end{bmatrix} \begin{bmatrix} V_{d1} \\ E_{DCI} \end{bmatrix} \tag{A.4}$$

Therefore the power injections would be,

$$\begin{bmatrix} S_{dl} \\ S_{0vl} \end{bmatrix} = \begin{bmatrix} V_{d1} \\ E_{DCI} \end{bmatrix} \times \begin{bmatrix} Y_1 \\ -k_2 m_{d1} \angle - \phi_1 Y_1 \\ -k_2 m_{d1} \angle \phi_1 Y_1 \end{bmatrix} \begin{bmatrix} Y_1 \\ -k_2 m_{d1}^2 (Y_1 + jB_{eq}) + G_{swl} \end{bmatrix} \begin{bmatrix} V_{d1} \\ E_{DCI} \end{bmatrix} \tag{A.5}$$

Appendix C.

As a first step, the implicit trapezoidal method calls for algebraizing any differential equation $y$ by means of expressing its step-by-step solution as an integral form,

$$\dot{y}(t) = F(X(t), Y(t)) = 0 \tag{B.1}$$

$$Y(t) - Y(t - \Delta t) - \int_{t - \Delta t}^{t} F(X(t), Y(t))dt = 0 \tag{B.2}$$

Assuming that all functions $F(\cdot)$ vary linearly over the time interval [$t - \Delta t$, $t$], the area under the integral can be approximated by a trapezium; the differential algebraic equation given in the form of a mismatch equation is then,

$$F_Y = Y_{t - \Delta t} + \frac{\Delta t}{2} Y_{t - \Delta t} - \left( Y_t - \frac{\Delta t}{2} Y_t \right) = 0 \tag{B.3}$$
Appendix D.

Parameters used in Section 4.1: (i) The parameters of the power system can be found in the section of ‘demos’ in Simulink® as: VSC-Based HVDC Transmission System (Detailed Model); (ii) VSC-HVDC data based on the HVDC rating $S_{nom} = 200$ MVA: $R_{DC} = 0.042704$ p.u.; $E_{DCnom} = 1.0$ p.u.; $G_{ij} = G_{ij} = 2e-3$ p.u.; $K_{trans} = 90$ p.u.; $X_{ij} = X_{ij} = 1e-3$ p.u.; $H_{c} = 0.014$; $K_{pedc} = 0.6$; $K_{pedc} = 35$; $K_{ppdc} = 0$; $K_{ppdc} = 5$; $K_{trans} = 25$; $T_{trans} = 0.02$; $Z_{LTCtransf} = 0.005 + j0.15$ p.u.

Parameters used in Section 4.2: (i) Synchronous generators are equipped with exciter, automatic voltage regulator, speed governor and hydro turbine. Generators and network data are available in [17]. (ii) VSC-HVDC data based on the HVDC rating $S_{nom} = 300$ MVA: $R_{DC} = 0.0024$ p.u.; $E_{DCnom} = 1.0$ p.u.; $G_{ij} = G_{ij} = 2e-3$ p.u.; $R_{ij} = R_{ij} = 2e-3$ p.u.; $X_{ij} = X_{ij} = 0.01$ p.u.; $H_{c} = 0.007$; $K_{pedc} = 0.05$; $K_{pedc} = 1.0$; $K_{ipdc} = 0.002$; $K_{ipdc} = 0.075$; $K_{trans} = 25.0$; $T_{trans} = 0.02$; $X_{LTCtransf} = 0.05$ p.u. (iii) Load models: $P_L = P_{LO} \left(0.2 + 0.4 \left(\frac{V}{V_0}\right) + 0.4 \left(\frac{V}{V_0}\right)^2\right)$ and $Q_L = Q_{LO} \left(0.2 + 0.4 \left(\frac{V}{V_0}\right) + 0.4 \left(\frac{V}{V_0}\right)^2\right)$, where, $P_{LO}$ and $Q_{LO}$ are the nominal active and reactive powers drawn by the load at rated voltage $V_0$.

References