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Parameter estimation of linear and nonlinear systems based on orthogonal series

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Abstract

A frequency domain based methodology using orthogonal series for estimation Linear systems and nonlinear systems is presented. The method can be used in the parameter identification of linear and nonlinear systems. The main advantage of using orthogonal functions is that integro-differentials equations are transformed into algebraic equations, this facilitates the identification process. The method presented in this paper was proved satisfactory to estimate parameters in two applications: An antenna azimuth position control system and a single phase transformer.

Keywords: Operational Matrix; Identification; Orthogonal series; linear systems; nonlinear systems.

1. Introduction

The design of robust and high performance industrial control systems depends critically on the accurate modeling and identification of the relevant process dynamics. The reason for this is that there is a need for operation of industrial processes in a very predictable way in order to meet demands such as reliable and profitable operation under quality and flexibility constraints. A detailed knowledge of the relevant process dynamics in the form of a mathematical model, and its use for operation and model control, is crucial for meeting these demands [1].

In general, linear and nonlinear systems identification can be divided into two categories, parametric or non-parametric. If the structure of the unknown systems is available a priori, identification becomes a parameter estimation problem. Though non-trivial, it is essentially a nonlinear optimization problem. In this way, either a black box, gray box or white box approach can be used depending whether a general ready-made model description or tailor made model set incorporating a priori information is utilized [2,3]. On the other hand, if little a priori information is available on the structure, the identification process is non-parametric, which is usually a much harder problem. In this paper, the problem of identification of parameter systems is addressed in a more general form. The method presented is an alternative to estimate parameters for which a dynamic model is known [4,5,6].

A distinctive feature of the proposed approach is the use of the orthogonal series. This allows writing a set of linear algebraic equations that can be solved using a pseudoinverse to obtain the unknown parameters.

The use of orthogonal series expansions is well known as an alternative for approximation and representation of functions. During the last two decades algebraic methods have been established for the solution of described by differential equations, such as analysis of linear time invariant and time varying systems, model reduction; optimal
control and system identification. The problem of parameter identification using orthogonal series includes linear time varying/invariant lumped and distributed systems, and nonlinear systems.

Transformations between different coordinate systems are popular in engineering calculations between different systems of coordinates are popular in engineering calculations. Mathematicians who first proposed these domains referred to them as ‘images’ (in French) because the transformed functions were maps or images of the original (usually time) functions. It can be said that the various domains cited are alternative ‘windows’ or ‘images’ to view a problem, and one wishes to employ the window or image that gives the best view of the problem and the most efficient calculation. Although solutions are the same in all domains, the bandwidth of the problem and numerical characteristics of intermediate matrices used are usually different, e.g. the set of equations of a particular problem may be sparser (or fewer) in one domain than in another. Typical examples are the applications of Laguerre polynomials, Legendre polynomials, Chebyshev polynomials of the first and second kind, Fourier series, Walsh series, block-pulse series, Haar series and Hartley series [4-10].

The main purpose of this paper is to provide an alternative method for parametric identification is proposed. It is based on the transformation the integro-differential equations into algebraic equations, this facilitates the identification process. Another advantages of the method are: easy implementation of the algorithm (MatLab®, Mathematica®, C code, etc.) and the easy control of the identification procedure and algorithm.

Section 2 is dedicated to the general background on approximation using orthogonal series expansions and operational matrices. In section 3 the algorithm for the parametric identification for linear and nonlinear systems is presented. The test cases can be found in Section 4. Finally some conclusions are shown.

2. Orthogonal Series Expansions and Operational Properties

In this section general background on approximation using orthogonal series expansions and operational matrices that is relevant for the material that follows is presented. The notation of presentation is kept general in order to include all possible domains. With exception of some mathematical curiosities, most time functions of common engineering interest \( f(t) \) may be approximated to arbitrary accuracy using orthogonal series expansions. The common feature of orthogonal series is that the basis functions used in the series are orthogonal to each other, that is when a pair of basis functions is integrated, the integral is zero if the two functions are different, and nonzero if they are the same. This property is used to find the coefficients of the series easily. Orthogonal series expansions also share other properties known as operational properties. In spite of having many common properties, orthogonal series expansions are different in their respective kernel or basis functions. While the kernel function of a complex Fourier series is \( e^{-j\omega t} \), the Hartley series utilizes \( \cos(\omega t) + \sin(\omega t) \), also known as the cosine-and-sine function or \( \text{cas}(\omega t) \). There are orthogonal series expansions where the kernels are step functions such as Walsh series, block-pulse series or the basic Haar wavelet. Some kernels are aperiodic, some are transient, and some are polynomial in form. The challenge is to select a kernel that results in low computation, few terms in the series representation of \( f(t) \), and low error.

If the vector of basis functions, \( T(t) \), is used to include a particular set of basis kernels and denoted as (note that the prime notation indenticates transposition),

\[
\Phi(t)=[\phi_n, \cdots, \phi_1, \phi_0, \phi_1, \cdots, \phi_n]
\]

Thus a function can be approximated as

\[
f(t) = c^\top \Phi(t)
\]

\[
c = [c_n, \cdots, c_1, c_0, c_1, \cdots, c_n]
\]

Each coefficient of vector \( c \) is calculated using inner product in equation (4)

\[
c_n = \frac{1}{T_0} \int_0^{T_0} f(t) \Phi(t) dt
\]

The operational properties of the orthogonal series may be written in terms of operational matrices of integration and differentiation. The main concept around these properties is the fact that the integral of an orthogonal series may be also expressed as a orthogonal series. The same can be stated for orthogonal series and their derivatives. In general terms the operational matrix of integration and differentiation may be defined as

\[
\int_0^t \cdots \int_0^t \Phi(t) d\tau = P^n \Phi(t)
\]
\[
\frac{d^n \Phi(t)}{dt^n} = D^n \Phi(t)
\]  
(6)

Note that \(P\) and \(D\) are operational matrices of integration and differentiation respectively.

At this point is also convenient to define two additional important matrices that may be derived also from properties of orthogonal series expansions. It is important to say that these matrices are not referred as operational matrices because they are not related to the integration and differentiation operators but they are nevertheless related to the multiplication operator. The two additional matrices are the product matrix \(A\) the matrix of coefficients and they are defined as follows. By the definition, the product of the orthogonal series basis vector and its transpose is called matrix product, \(\Pi(t)\), namely

\[
\Pi(t) = \Phi(t) \Phi(t)\]
(7)

The matrix of coefficients is defined in terms of the product matrix and the orthogonal series basis vector as a matrix that satisfies,

\[
\Pi(t) c = [C] \Phi(t)
\]
(8)

Matrix \([C]\) is the matrix of coefficients given in vector \(c\). Interesting to say, is the fact that the product and coefficient matrix are indeed a properties that made possible analytical solutions for time varying systems and that they were found for the first time in the Walsh domain. In general, algebraic methods of operational matrices are a unified approach independent of the type of orthogonal series used to solve a problem [3],[8],[9]. The particularities of each domain are reflected in the numerical characteristics of matrices \(P\), \(D\), \(\Pi(t)\), \([C]\) and vector of coefficients \(c\). Structures of \(P\), \(D\), \(\Pi(t)\), and \([C]\) of different domains varies. The sparsity of the matrices is also different which make some formulations more efficient than others for applications such as optimal control, system analysis, and on-line calculations. In terms of approximations, the selection of domain is also relevant.

3. Parameter estimation for Linear and Nonlinear Systems

Any given differential equation of degree \(n\) can be expressed as a set of state-space equations. This same set can be transformed into an algebraic set of equations using operational matrices, in this section a methodology to solve the identification problem and estimate the unknown parameters using the input-output data is presented [3].

3.1. Parameter estimation for Linear Systems

Considering as a general case of linear systems, the system described by state-space equations,

\[
\dot{x}(t) = A(t)x(t) + B(t)u(t)
\]
(9)

where \(x(t) \in \mathbb{R}^{n\times 1}\) and \(u(t) \in \mathbb{R}^{p\times 1}\) are the state and input vectors respectively, and the corresponding time-varying coefficients \(A(t) \in \mathbb{R}^{n \times n}\) and \(B(t) \in \mathbb{R}^{n \times p}\).

Using approximations via orthogonal series expressions the elements \(a_{ij}(t)\) and \(b_{ij}(t)\) of matrices \(A(t) \in \mathbb{R}^{n \times n}\) and \(B(t) \in \mathbb{R}^{n \times p}\) that satisfy the Dirichlet conditions in the interval \((0,1)\), can be expressed as,

\[
a_{ij}(t) \equiv A_{ij} \Phi(t)
\]
(10)

\[
b_{ij}(t) \equiv B_{ij} \Phi(t)
\]
(11)

Where

\[
A_{ij} = [A_{i,j,0} \ldots A_{i,j,n}]
\]

\[
B_{ij} = [B_{i,j,0} \ldots B_{i,j,n}]
\]

Similarly, we have the elements of vector \(x(t) \in \mathbb{R}^{n\times 1}\) and \(u(t) \in \mathbb{R}^{p\times 1}\) can be approximated as,

\[
x(t) \equiv X(t) \Phi(t)
\]
(12)
\[ A(t)x(t) = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_n(t) \end{bmatrix} \Phi(t) \]

(13)

Where every factor \( A_y(t)X(t)T(t) \) may be rearranged using the product and coefficient matrix as follows,

\[ A_y(t)X(t)\Phi(t) = A_y(t)\Phi(t)X(t) = A_y(t)[X(t)]\Phi(t) \]

(14)

Matrix \([X_j(t)]\in \mathbb{R}^{n(m)}\) is the coefficient matrix of the vector \( x(t)\in \mathbb{R}^{n(\text{nd})} \). It may be shown that with the above simplification the product \( A(t)x(t) \) may be written as,

\[ A(t)x(t) = A[X]\Phi(t) \]

(15)

In other words,

\[ A(t)x(t) = A[X]\Phi(t) \]

(16)

Where \([X_j(t)]\) is the coefficient matrix corresponding to each coefficient, \([X]\) is the matrix formed by all the coefficient matrices \([X_j(t)]\). \( A \) is the matrix formed for every vector \( A_y(t) \) and \( \Phi(t) \) is the basis vector.

A similar procedure is used for \( B(t)u(t) \) yielding,

\[ B(t)u(t) = B[U]\Phi(t) \]

(17)

Where \( B(t)\in \mathbb{R}^{n(q(m))}, [U]\in \mathbb{R}^{q(m)} \) and \( \Phi(t)\in \mathbb{R}^{n(\text{nd})} \).

Integrating in both sides (9), we have that,

\[ x(t) - x_0(t) = \int_0^t A(t)x(t)dt + \int_0^t B(t)u(t)dt \]

(18)

Using operational matrices and substituting (21) and (22) into (23) yields,

\[ X\Phi(t) = x_0(t)\Phi(t) + A[X]P\Phi(t) + B[U]P\Phi(t) \]

(19)

Where \( x_0(t) \) is the same size of \( X \) and contains the initial conditions of every state.

\[ x_0(t) = \begin{bmatrix} x_1(0) & 0 & \cdots & 0 \\ x_2(0) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ x_n(0) & 0 & \cdots & 0 \end{bmatrix} \]

(20)

Equation (19) can be rewritten as,

\[ X = \Theta Z \]

(21)
Where

\[ \theta = \begin{bmatrix} A & B & X_0 \end{bmatrix} \]  
\[ Z = \begin{bmatrix} [X]^P \\ [U]^P \\ e \end{bmatrix} \]  
\[ e = [1 \ 0 \ 0 \ 0 \ \cdots] \]  

Using the above equations and solving for unknown parameters we obtain [3]:

\[ \theta = XZ (ZZ')^{-1} \]  

From the equation (25) the parameter coefficients can be obtained provided that \( (ZZ')^{-1} \) exists.

### 3.2. Correlation Coefficient

With the only purpose of validating the obtained model, a set of different criteria can be used. One of those is the correlation coefficient, used to obtain input to output validation. The application of this criteria requires the simulation of the obtained model with \( \hat{\theta} \), obtaining measurements of every state variable in the system and applying the expression,

\[ r = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \]

where \( y \) is the signal obtained from the physical system, and \( \bar{y} \) it is the arithmetic average of \( y \) and \( \hat{y} \) is the signal obtained by the simulation model. The correlation coefficient indicates what proportion of the total variation of \( y \) is explained by the identified model. For example, if the correlation factor is \( r = 0.95 \), this means that the 95% of the variation of \( y \) is described with the identified model. In the following table, the different levels of the correlation coefficient are shown, depending on its value [11].

<table>
<thead>
<tr>
<th>Correlation Coefficient</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 1 )</td>
<td>Perfect</td>
</tr>
<tr>
<td>0.90 ( \leq r \leq 0.99 )</td>
<td>Very Good</td>
</tr>
<tr>
<td>0.80 ( \leq r \leq 0.89 )</td>
<td>Good</td>
</tr>
<tr>
<td>( r \leq 0.79 )</td>
<td>Regular</td>
</tr>
</tbody>
</table>

### 3.3. Parameter Estimation for Nonlinear Systems

This section argues that the parametric estimation methodology shown in section 3.2 can be extended to nonlinear systems. In order to achieve this, the problem of identifying the nonlinearity becomes the problem of identifying the coefficients of the orthogonal series that approximate the unknown nonlinearity [11]. The methodology is illustrated with the following example.

Consider the model of the mechanical spring-mass system shown in Fig. 1, where \( FSP \), which is the force associated with the spring is a nonlinear function of the displacement. To achieve these an important assumption must be made; the unknown analytical nonlinear can be represented by an orthogonal series. If \( x_1(t) = y(t) \) and \( x_2(t) = y(t) \) are selected as state variables, the system can be modeled as:

\[ \begin{align*}
\cdot & x_1 = x_2 \\
\cdot & x_1 = -\frac{k}{m} x_1 - \frac{c}{m} x_2 - \frac{ka^2}{m} x_1^3 + \frac{1}{m} A \cos \omega t
\end{align*} \]
From equation (27) we can see that the term \( \frac{k_x a^3}{m} x_1^3 \) which is a nonlinear part of the equation that could be generally function represented as \( f(x_i) \). The system input \( f(t) \) can be considered as \( u = A \cos \omega_t \). With this information the state space system can be written as follows,

\[
\begin{align*}
    x_1 &= a_1 x_2 \\
    x_2 &= a_3 x_1 + a_{23} x_2 + f(x_i) + a_{2} u \\
\end{align*}
\]  

(28)

where constants \( a_{11}, a_{21}, a_{23} \) and \( a_2 \) represent the parameters that must be identified. In this case, it is assumed that the form of \( f(x_i) \) is unknown, in other words, the objective is to identify the form of this nonlinear function by an approximation using the trigonometric Fourier series,

\[ f(x_i) = \frac{a_0}{2} + \sum_{m=1}^{r} a_i \cos(ix_i) + b_i \sin(ix_i) \] 

(29)

Because of the fact that parametric identification is realized by measuring the system response and the input, the function values \( \cos(ix_i) \) and \( \sin(ix_i) \) can be calculated directly, this can be done because we have measurements of the state \( x_i \) and the unknown parameters are the \( a_i, \) and \( b_i \) that are related to the coefficients included in the identification methodology.

An additional assumption that can be made in order to improve the identification process is derived from the practical observation that the most common nonlinear forms are functions with odd symmetry with respect to the origin, where the following condition is met,

\[ f(x) = -f(-x) \] 

(30)

This fact can be used in the benefit of the estimation process. For instances, the Fourier coefficients \( a_i \) do not exist, with \( i = 0, 1, 2, \ldots, r \). Then, the approximation of \( f(x_i) \) must be obtained with the following expression,

\[ f(x_i) = \sum_{i=1}^{r} b_i \sin(ix_i) \] 

(31)

this leads to the need of estimating only \( b_i \) coefficients. Thus, reordering (28) we obtain,

\[
\begin{align*}
    x_1 &= a_1 x_2 \\
    x_2 &= a_{21} x_1 + a_{23} x_2 + a_{2} u + b_1 \sin(x_1) + b_2 \sin(2x_1) + \ldots + b_r \sin(rx_1) \\
\end{align*}
\]  

(32)

Integrating the expressions given by (32), the following is obtained,

\[
\begin{align*}
    x_1 - x_1(0) &= a_1 \int_0^1 x_1 d\sigma \\
    x_2 - x_1(0) &= a_{21} \int_0^1 x_1 d\sigma + a_{23} \int_0^1 x_2 d\sigma + a_{2} \int_0^1 u d\sigma + b_1 \int_0^1 \sin(x_1) d\sigma + \ldots + b_r \int_0^1 \sin(rx_1) d\sigma \\
\end{align*}
\]  

(33, 34)
Then, a variable change is made using $z_r = \text{sen}(r_x)$.

\[ x_i - x_i(0) = a_{i2} \int_0^{r_x} x_i d\sigma \]  
\[ x_2 - x_2(0) = a_{21} \int_0^{r_x} x_1 d\sigma + a_{22} \int_0^{r_x} x_2 d\sigma + a_{23} \int \mu d\sigma + b_1 \int z_r d\sigma + \ldots + b_j \int z_r d\sigma \]

(35)  
(36)

Now, the approximation specified in (12) is carried out for $X_1$, $X_2$, $X_1(0)$, $X_2(0)$, $z_1$, …, $z_r$ yielding,

\[ X_i \Phi(t) - X_i^0 \Phi(t) = a_{i1} \int X_i \Phi(\sigma) d\sigma \]  
\[ X_2 \Phi(t) - X_2^0 \Phi(t) = a_{21} \int X_1 \Phi(\sigma) d\sigma + a_{22} \int X_2 \Phi(\sigma) d\sigma + a_{23} \int U \Phi(\sigma) d\sigma + b_1 \int Z_1 \Phi(\sigma) d\sigma + \ldots + b_j \int Z_r \Phi(\sigma) d\sigma \]

(37)  
(38)

Applying the integration operational matrix defined in (5), into equations (37) and (38) yields

\[ X_i \Phi(t) - X_i^0 \Phi(t) = a_{i1} X_2 P \Phi(t) \]

(39)

\[ X_2 \Phi(t) - X_2^0 \Phi(t) = a_{21} X_1 P \Phi(t) + a_{22} X_2 P \Phi(t) + a_{23} U P \Phi(t) + b_1 Z_1 P \Phi(t) + \cdots + b_r Z_r \Phi(t) \]

(40)

Finally, eliminating the time dependency, in (39) the system is transformed into a set of algebraic equations,

\[ X_1 - X_1^0 = a_{21} X_2 P \]

(41)

\[ X_2 - X_2^0 = a_{21} X_1 P + a_{22} X_2 P + a_{23} U P + b_1 Z_1 P + \cdots + b_r Z_r \]

(42)

Writing (41) and (42) in matrix vector notation results in, note that the result is written in terms of the coefficients of the Fourier series,

\[
\begin{bmatrix}
X_1 - X_1^0 \\
X_2 - X_2^0
\end{bmatrix} =
\begin{bmatrix}
0 & a_{i1} & 0 & \ldots & 0 \\
a_{21} & a_{22} & a_{23} & \ldots & b_j
\end{bmatrix}
\begin{bmatrix}
X_1 P \\
X_2 P \\
U P \\
Z_1 P \\
Z_r P
\end{bmatrix}
\]

(43)

Or in compact form

\[ X = \Theta \cdot M \]

(44)

The aforementioned expression has a solution,

\[ \hat{\Theta} = X \cdot M^t \left(M \cdot M^t \right)^{-1} \]

(45)

4. Simulation Results

4.1. Test case: Parameter estimation for linear systems

To illustrate the identification approach described in this paper a time invariant linear systems is considered first. An antenna azimuth position control system is used. The dynamic of the system in Figure 2 is represented by [12],

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & -1/J_w \left(D_w + \frac{K_z}{R_w} \right)
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{K_z}{R_w}
\end{bmatrix} e_w(t)
\]

(46)

Where $J_w = 0.03 \text{ kg-m}^2$, $D_w = 0.02 \text{ N-m s/rad}$, $\frac{K_z}{R_w} = 0.0625 \text{ N-m/A} \Omega$, $K_b = 0.5 \text{ V-s/rad}$. Substituting the values into Equation (46), we obtain the final state-space representation:
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1.71 & 2.083 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} e_i(t)
\]
(47)

As it has been mentioned before the mathematical description in this paper is rather general. The selection of a particular orthogonal series is left to the analyst’s preference, in this the Walsh series based are considered in this test case, orthogonal series expansions and operational properties via Walsh series are described in [3].

Table 2 shows the results of the estimated parameters and errors for each parameter. In this case the accuracy is obtained by using the equation (45) and 128 Walsh basis functions.

**Table 2. Results of parameter estimation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Exact value</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₁₁</td>
<td>0</td>
<td>6.4x10⁻⁷</td>
</tr>
<tr>
<td>a₁₂</td>
<td>1</td>
<td>0.9995</td>
</tr>
<tr>
<td>a₂₁</td>
<td>0</td>
<td>-7.52x10⁻⁶</td>
</tr>
<tr>
<td>a₂₂</td>
<td>-1.71</td>
<td>-1.636</td>
</tr>
<tr>
<td>b₁₁</td>
<td>0</td>
<td>5.81x10⁻⁶</td>
</tr>
<tr>
<td>b₂₁</td>
<td>2.083</td>
<td>1.993</td>
</tr>
</tbody>
</table>

Figure 3 a) and 3 b) shows the system’s output to a step input. The response was obtained by comparison the system’s output using the estimated parameter vs real parameter. It is possible to see a closed behavior between the actual system and the identified one for both the state x₁ (angular position) as the state x₂ (angular speed).

**4.2. Test case: Parameter identification for nonlinear systems**

Normally, for theoretical applications, transformers can be expressed as linear systems mainly because the analysis is based on nominal conditions. Nevertheless, in practical applications, electrical transformers are actually nonlinear systems due to the fact of the hysteresis phenomena at the transformer core. Therefore it is important to estimate the parameters in the model under these circumstances. In this example our methodology is applied to estimate the nonlinear characteristic. A schematic showing the nonlinear model of a single-phase transformer is shown in Figure 4. The overall structure of the model is obtained from [11].
The following equations (48) and (49) describe a model of a single-phase transformer,

\[
\frac{di(t)}{dt} = \frac{1}{l_1} v_{r1}(t) + \frac{r_0}{l_1} i(t) + \frac{r_1}{l_1} f(\psi(t))
\]

(48)

\[
\frac{d\psi(t)}{dt} = r_1 j(t) - r_0 f(\psi(t))
\]

(49)

The nonlinear characteristic is a flux dependent time function \( f(\psi(t)) \). The real parameters in the transformer are \( r_1 = 0.192 \Omega \), \( r_0 = 612.86 \Omega \) and \( l_1 = 0.9 \text{mH} \). The nonlinear function is described by [11],

\[
f(\psi(t)) = 0.7576\psi(t) + 1.03 \times 10^7 \psi^9(t)
\]

(50)

With this information, the simulation can be done in order to measure the magnetic flux and the current. In this case, the input signal is \( v_p = 119 \cos(\omega t - 42.97^\circ) \) V, with \( f_0 = 60 \text{Hz} \).

Figures 5 a) and 5 b) show the evolution of ten cycles of the current and the magnetic flux through time respectively. These signals are used throughout the identification process.

In order to perform the identification of the transformer model, equations (51) and (52) are rewritten and the unknown nonlinear function is considered,

\[
\frac{di(t)}{dt} = a_1 i(t) + b_1 v_{r1}(t) + g(\psi(t))
\]

(51)

\[
\frac{d\psi(t)}{dt} = a_2 j(t) + h(\psi(t))
\]

(52)

It can be observed, from these equations, that the parameters that are going to be identified are \( a_1, a_2 \) and \( b_1 \) and also two nonlinear functions \( g(\psi) \) and \( h(\psi) \). As additional information of the system, it is known that the saturation effect can be represented by an odd symmetry function, which is going to be approximated only by odd functions.

Applying the nonlinear systems identification methodology, a total of 512 Haar coefficients and 11 Fourier functions were used in order to approximate the nonlinear function. Fig. 6 a) shows the identified nonlinear function \( g(\psi(t)) \) affected by the scaling factor \( \frac{l_1}{r_0} \). In Fig. 6 b), the function \( h(\psi(t)) \) scaled by \( -\frac{1}{r_0} \) is displayed. These signals are expected to be equal.
The parameters estimation using orthogonal functions Haar are shown in table 3,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Exact value</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>-681168.88</td>
<td>-680899.59</td>
</tr>
<tr>
<td>$a_2$</td>
<td>612.86</td>
<td>613.10</td>
</tr>
<tr>
<td>$b_1$</td>
<td>1111.11</td>
<td>1110.67</td>
</tr>
</tbody>
</table>

Figure 7 shows the evolution of the two nonlinear identified functions and the nonlinear function described by (50), where there is no difference between the three functions, the correlation factor is $r = 0.9584$ for the current and the correlation factor magnetic flux is $r = 1.00061$, this mean a very good accuracy for the parametric identification.

5. Conclusions

In this paper the properties of the orthogonal series together with the operational matrices were used for parameter estimation of linear and nonlinear systems. The method use a general expression in the frequency domain to estimate parameters based in orthogonal series. The orthogonal series and some useful properties such as the operational matrix of the integration, product and coefficient matrices are presented. These properties are applied to estimate parameters in two applications: An antenna azimuth position control system and a single phase transformer. In both cases the method attains accurate results, as it is shown in the comparison of the response of the actual systems and the estimated system. Clearly, better results are dependent on the number of coefficients in the approximation.

References